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CERTIFIED INVESTMENT RESEARCH ANALYST®

Multi factor risk and return Models

01 CAPM

02 Security Market Line (SML)

03 Multi-Factor Model of Risk & Return

CAPM Theory

Key Learning Outcome

- Introduction
- CAPM pricing model

Introduction

- In 1964, Sharpe introduced the capital asset pricing model (CAPM), a model for the expected return of assets in equilibrium based on a mean– variance foundation.
- CAPM is an excellent starting point to portfolio theory. It builds on the Markowitz mean–variance-efficiency model.
- Risk-averse investors with a one-period horizon care only about expected returns and the variance of returns (risk).
- These investors choose only efficient portfolios, with minimum variance given expected return, and maximum expected return, given variance.

CAPM (Capital Asset Pricing Model)

The CAPM is a logical consequence of mean–variance portfolio theory and assumes:

- All investors have homogeneous expectations;
- Investors choose their risky asset proportions by maximising the Sharpe ratio;
- Investors can borrow or lend unlimited amounts at the risk-free rate;
- The market is in equilibrium at all times.

CAPM (Variance, Co-Variance, Correlation)

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i)$$

$$\sigma_p^2 = \sum_{i=1,j=1}^{N} w_i w_j CoV(i,j)$$

$$\sum_{i=1}^{N} w_i = 1$$

$$Given: Cov(i,j) = \rho_{ij}\sigma_i\sigma_j \ and \ CoV\ (i,i) = \sigma_i^2$$

$$Then: \sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i,j=1,i\neq j}^{N} w_i w_j \rho_{ij}\sigma_i\sigma_j$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

CAPM: Portfolio Risk and Return

Portfolio	Weight in Asset 1	Weight in Asset 2	Portfolio Return	Portfolio Standard Deviation
X	25.0%	75.0%	6.25%	9.01%
Y	50.0	50.0	7.50	11.18
Z	75.0	25.0	8.75	15.21
Return	10.0%	5.0%		
Standard deviation	20.0%	10.0%		
Correlation between Assets 1 and 2		0.0		

$$O_X = \sqrt{(.25)^2(.20)^2 + (.75)^2(.10)^2 + (.25)(0)(.20)(.10) + (.75)(0)(.10)(.20)} \approx 9.01\%$$

CAPM: Calculation and Interpretation of Beta

$$\beta_i = \frac{CoV(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

$$\beta_i = \frac{0.026250}{0.02250} = \frac{0.70 \times 0.25 \times 0.15}{0.02250} = \frac{0.70 \times 0.25}{0.15} = 1.17$$

Market's Return → **Asset Beta** → **Asset's Return**

If an asset's covariance with the market and market variance are given as 0.026250 and 0.02250, respectively, the calculation would be 0.026250/0.02250 = 1.17. If the correlation between an asset and the market is 0.70 and the asset and market have standard deviations of return of 0.25 and 0.15, respectively, the asset's beta would be (0.70)(0.25)/0.15 = 1.17.

CAPM (Capital Asset Pricing Model)

Equation to determine the return required by investors to willingly hold any particular risky asset (as part of a well-diversified portfolio):

Required return on asset i = Risk-free rate + Risk premium

$$ER_i = r + \beta_i (ER_m - r)$$

Where

β_i = the asset's beta. The 'risk premium', therefore, consists of the market risk premium (ER_m - r) multiplied by 'beta', referred to as the 'price of market risk'

Portfolio Beta & Expected Return

Portfolio beta is the weighted sum of the betas of the component securities:

$$\beta_p = \sum_{i=1}^{N} w_i \beta_i = (0.40 \times 1.50) + (0.60 \times 1.20) = 1.32$$

The portfolio's expected return given by the CAPM is:

$$E(R_p) = R_f + \beta_p \left[E(R_m) - R_f \right]$$

$$E(R_p) = 3\% + 1.32[9\% - 3\%] = 10.92\%$$

Q: Beta of 2 securities are 1.5 and 1.2, and weights in portfolio 40% and 60% respectively. Calculate portfolio return if Rf=3%, Market Return=9%

CAPM (Capital Asset Pricing Model)

Beta is the primary determinant of expected return

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f \right]$$

$$E(R_i) = 3\% + 1.5\left[9\% - 3\%\right] = 12.0\%$$

$$E(R_i) = 3\% + 1.0\left[9\% - 3\%\right] = 9.0\%$$

CAPM Assumptions

- Investors are risk-averse, utility-maximizing, rational individuals.
- Markets are frictionless, including no transaction costs or taxes.
- Investors plan for the same single holding period.
- Investors have homogeneous expectations or beliefs. All investments are infinitely divisible.
- Investors are price takers.

Limitations of CAPM

Theoretical

- Single-factor model
- Single-period model

Practical

- Market portfolio
- Proxy for a market portfolio
- Estimation of beta
- Poor predictor of returns
- Homogeneity in investor expectations

Security Market Line

Key Learning Outcome

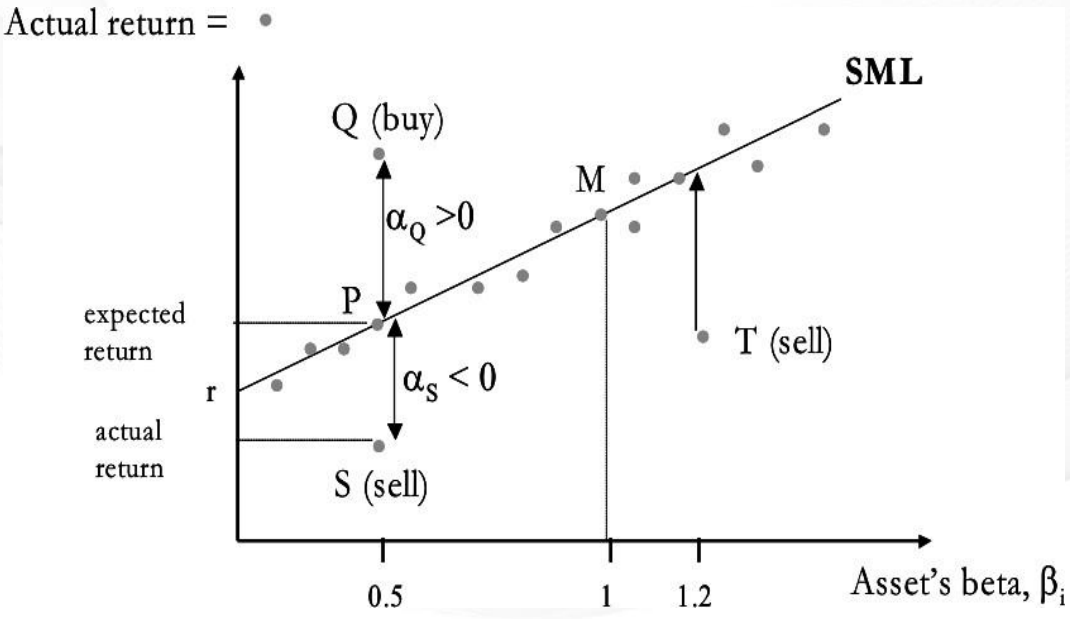
- Introduction
- Usage in Security Selection

Security Market Return

- The security market line (SML) is a line drawn on a chart that serves as a graphical representation of the capital asset pricing model (CAPM).
- The SML can help to determine whether an investment product would offer a favorable expected return compared to its level of risk.
- The formula for plotting the SML:

Required Return = Risk-Free Rate of Return + Beta (Market Return - Risk-Free Rate of Return).

Security Market Line

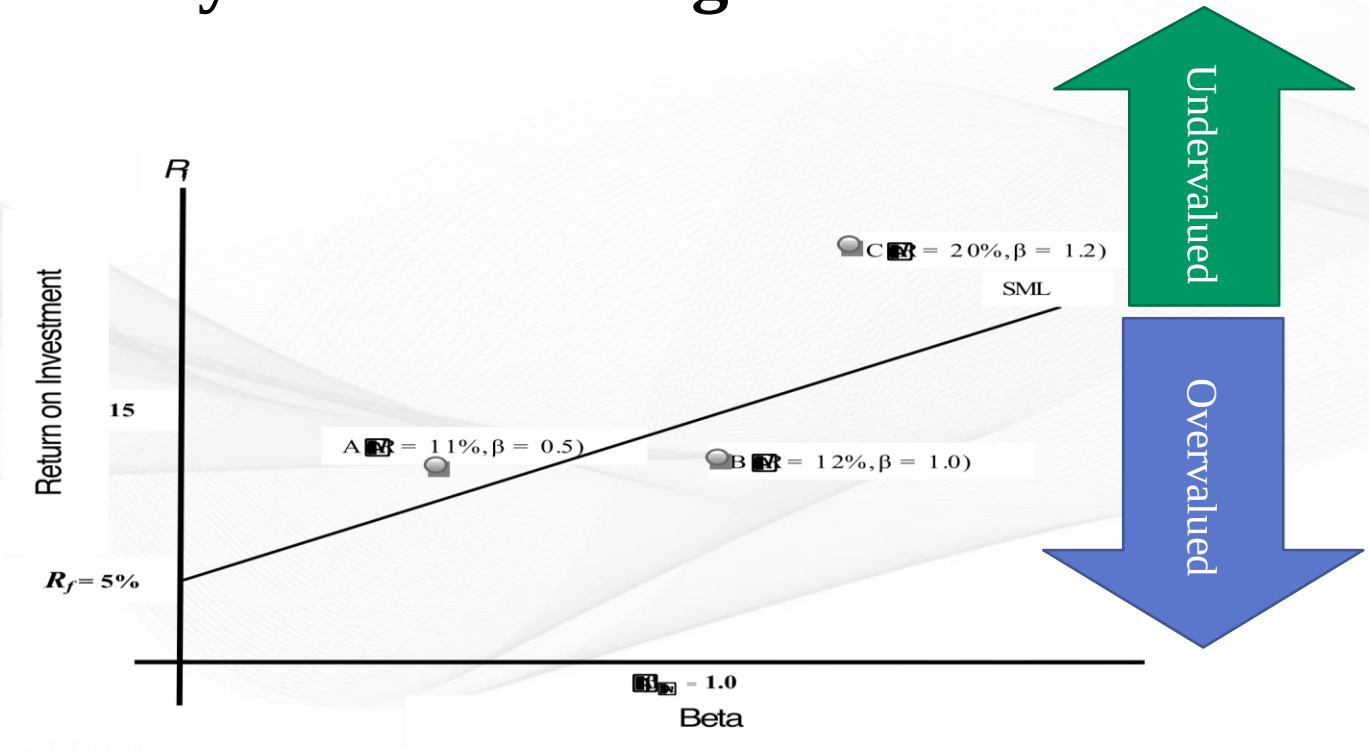


Required return = SML

Securities which lie above (below) the SML have a positive (negative) 'alpha'

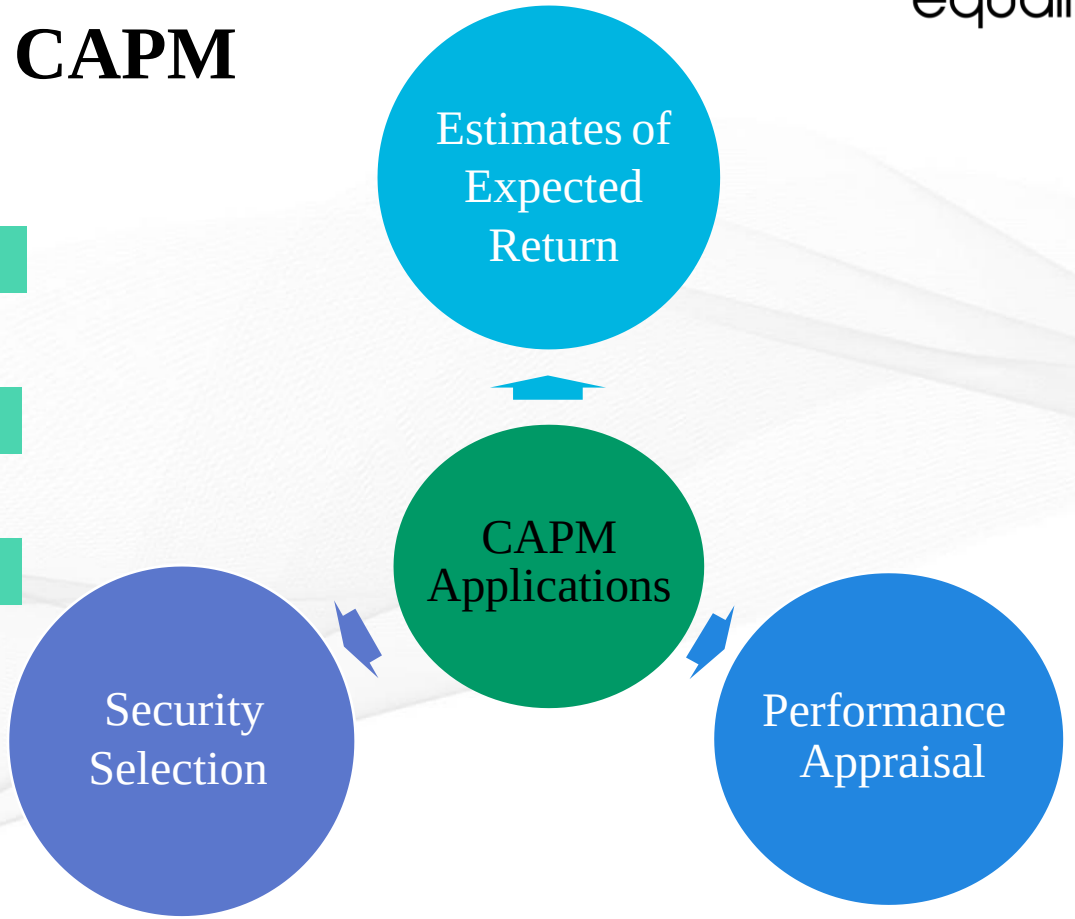
indicating an 'abnormal return' after correcting for 'beta risk'.

Security Selection using SML



Applications of the CAPM

- Performance Measurement
- Performance Appraisal
- Manager Selection



Multi-Factor Model

Key Learning Outcome

- Multi Factor Model
- Arbitrage Pricing Theory

Multi-Factor Models

A multi-factor model is a financial model that employs multiple factors in its calculations to explain market phenomena and/or equilibrium asset prices. The multi-factor model can be used to explain either an individual security or a portfolio of securities. It does so by comparing two or more factors to analyse relationships between variables and the resulting performance.

- Multi-factor portfolios are a financial modeling strategy in which multiple factors, macroeconomic as well as fundamental and statistical, are used to analyze and explain asset prices.
- The portfolios can be constructed using various methods: intersectional, combinational, and sequential modeling.

Arbitrage Pricing Theory (APT)

- Arbitrage pricing theory (APT) is a multi-factor asset pricing model based on the idea that an asset's returns can be predicted using the linear relationship between the asset’s expected return and a number of macroeconomic variables that capture systematic risk.
- Unlike the CAPM, which assume markets are perfectly efficient, APT assumes markets sometimes misprice securities, before the market eventually corrects and securities move back to fair value.
- Using APT, arbitrageurs hope to take advantage of any deviations from fair market value.

Four Factor Model

Systematic Risk

Value Anomaly

$$E(R_{it}) = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,UMD} UMD_t$$

Size Anomaly

Momentum Anomaly

Arbitrage Pricing Theory (APT) - Example

For example, the following four factors have been identified as explaining a stock's return and its sensitivity to each factor and the risk premium associated with each factor have been calculated:

- Gross domestic product (GDP) growth: ß = 0.6, RP = 4%
- Inflation rate: ß = 0.8, RP = 2%
- Gold prices: ß = -0.7, RP = 5%
- Standard and Poor's 500 index return: ß = 1.3, RP = 9%
- The risk-free rate is 3%

Using the APT formula, the expected return is calculated as:

$$R_{it} = a_1 + b_{i1}F_{1t} + b_{i2}F_{2t} + b_{i3}F_{3t} + \varepsilon_{it}$$

Expected return = 3% + (0.6 x 4%) + (0.8 x 2%) + (-0.7 x 5%) + (1.3 x 9%) = 15.2%

Key Takeaways

- Portfolio Beta and Expected return
- Return-generating models and the market model
- Capital asset pricing model (CAPM)
- Security Market Line (SML)
- Arbitrage Pricing Theory (APT) and Multi-factor models

Further Reading

- CAPM : https://www.cfainstitute.org/research/cfa-digest/2005/05/the-capital-asset-pricing-model-theory-and-evidence-digest-summary

- Security Market Line: https://www.investopedia.com/terms/s/sml.asp#:~:text=The%20security%20market%20line%20(SML,market%20at%20any%20given%20time.

- Multi Factor Models : https://www.investopedia.com/terms/m/multifactor-_model.asp

- APT: https://www.investopedia.com/terms/a/apt.asp

Thank You