



Bond Valuation

Introduction to Bonds

Key Learning Outcome

- Characteristics
- Legal & Regulatory Features
- Contingency Provisions

INTRODUCTION: What is a Bond (Fixed Income Security)?

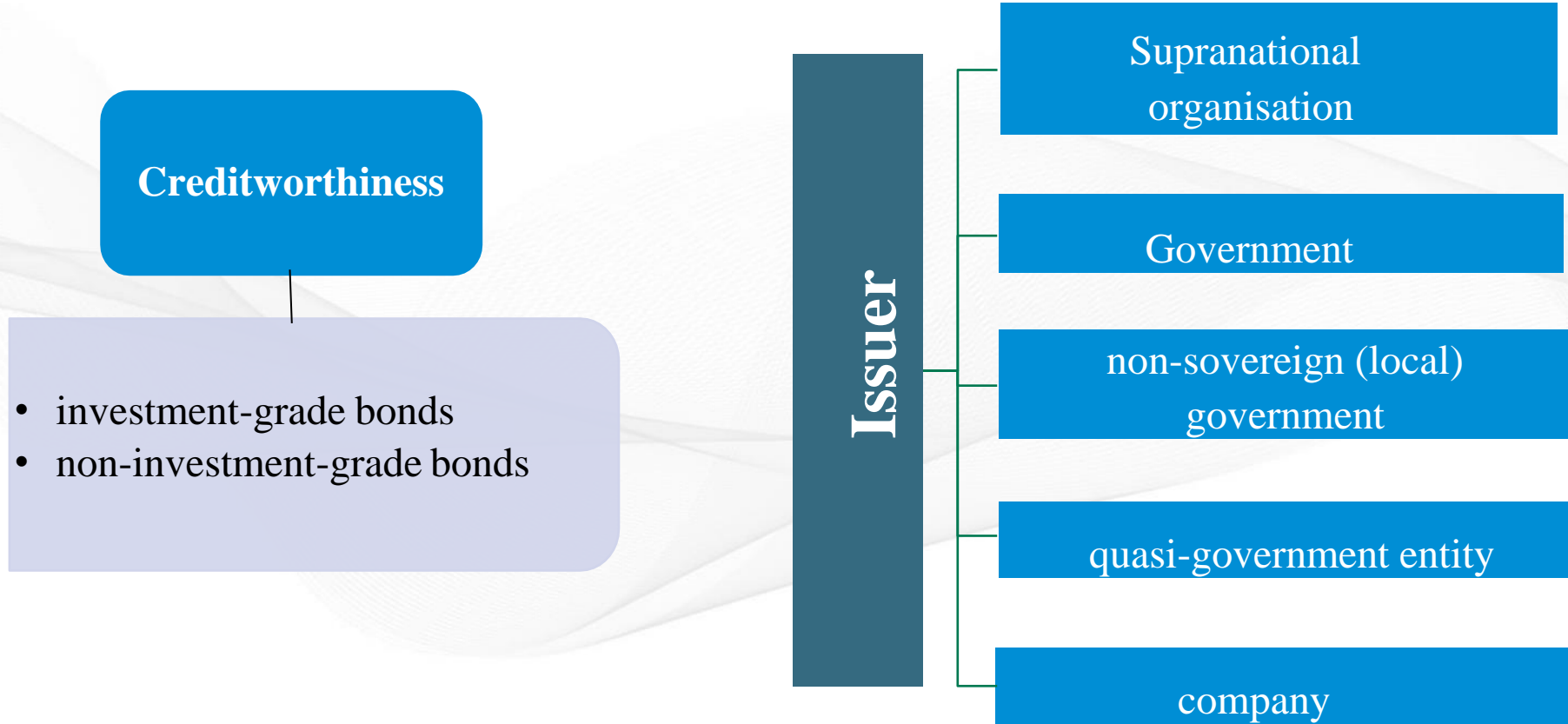
- A fixed-income security is a financial obligation of an entity (the issuer) that promises to pay a specified sum of money at specified future dates.
- A fixed-income security is an instrument that allow governments, companies, and other types of issuers to borrow money from investors.
- Any borrowing of money is debt.
- The terms “fixed-income securities,” “debt securities,” and “bonds” are often used interchangeably.

Bond: Characteristics and Markets

There are three important elements when investing in a fixed- income securities:

- The bond features, including the issuer, maturity, value, coupon rate and frequency, and currency denomination.
- The legal, regulatory, and tax considerations.
- The contingency provisions that may affect the bond's scheduled cash flows.

Fixed Income Securities: Issuers



Features of Fixed Income Securities

Maturity

- The maturity date is the date when the issuer is obligated to redeem the bond.
- The tenor, also known as term to maturity, is the time remaining until the bond's maturity date.
 - Money market securities are fixed-income securities with maturity up to one year.
 - Capital market securities are fixed-income securities with maturity longer than one year.

Par value (principal) of a Bond

- The par value of a bond is the amount the issuer agrees to repay the bondholders on the maturity date.

Features of Fixed Income Securities

Coupon Rate and Frequency

- The coupon or nominal rate (yield) of a bond is the interest rate that the issuer agrees to pay each year until the maturity date.
- The coupon is the annual amount of interest payments and is determined by multiplying the coupon rate by the par value of the bond.
 - Plain vanilla bonds pay a fixed rate of interest.
 - Floating-rate notes (FRNs) or floaters pay a floating rate: a reference rate plus a spread.
 - Bonds that do not pay interest are called “zero-coupon bonds.”

Conventional bonds pay a fixed periodic coupon over a specified time to maturity, typically annually or semi-annually and occasionally quarterly.

Instruments with other coupon structures:

- floating-rate notes
- step-up coupon bonds
- credit-linked coupon bonds
- deferred coupon bonds
- index-linked bonds

Legal, Regulatory and Tax Considerations

Bond indenture

- The trust deed is the legal contract that describes the form of the bond, the obligations of the issuer, and the rights of the bondholders.
- This legal contract is often called the “bond indenture.”
- The indenture is written in the name of the issuer and references features of the bond issue, such as par value, coupon rate and frequency, maturity date, and the funding sources for the interest and principal repayments, as well as any collaterals, covenants, and credit enhancements.

Bonds with contingency provisions

- A contingency provision is a clause in a legal document that allows for some action if the event or circumstance does occur (i.e., embedded option).
- Some common types of bonds with embedded options include callable bonds, puttable bonds, and convertible bonds.



Tax considerations

- Interest payments and capital gains are often subject to taxation. Tax treatment of both varies from jurisdiction to jurisdiction.
- The income portion of a bond investment is typically taxed at the ordinary income tax rate. Tax-exempt securities are the exception to this rule.
- A tax on capital gains may apply if the bond sale price exceeds the bond purchase price.
- The original issue discount might be subject to a tax for discount bonds (such as zero-coupon bonds).

Bonds with contingency provisions

- A contingency provision is a clause in a legal document that allows for some action if the event or circumstance does occur (i.e., embedded option).
- Some common types of bonds with embedded options include callable bonds, puttable bonds, and convertible bonds.

Callable bonds give the issuer the right to redeem all or part of the bond before the specified maturity date. The primary reason why issuers choose to issue callable bonds rather than non-callable bonds is to protect themselves against a decline in interest rates.

Puttable Bonds: The bondholder has the right to sell the bond back to the issuer at a pre-determined price on specified dates.

Puttable bonds are beneficial for the bondholder by guaranteeing a pre-specified selling price at the redemption dates.

Convertible Bonds: Hybrid security with both debt and equity features.

The bondholder has the right to exchange the bond for a specified number of common shares in the issuing company, and thus participate in the equity up side.

At the same time, the bondholder receives downside protection; if the share price does not appreciate, the convertible bond offers the comfort of regular coupon payments and the promise of principal repayment at maturity.



Bond Value Theorems

Key Learning Outcome

- Bond Yield & Yield to Maturity
- Term Structures & Interest Rates
- Duration & Convexity

Yield To Maturity

- Yield to maturity (YTM) is the total return anticipated on a bond if the bond is held until it matures.
- The internal rate of return (IRR) of an investment in a bond if the investor holds the bond until maturity, with all payments made as scheduled and reinvested at the same rate.
- Yield to maturity is also referred to as *Book yield* or *Redemption yield*.

YTM - Example

Scenario: An investor purchases a 10-year, 8% annual coupon bond at \$85.503075 per \$100 of par value and holds it to maturity.

The bond's yield to maturity is 10.40%. Show the sources of return:

Bondholder's Receipt

- 1) *Coupon payments* $10 \times \$8 = \80 ;
- 2) *Par value at maturity* \$100;
- 3) *Reinvestment income from coupons (at 10.40%)*.

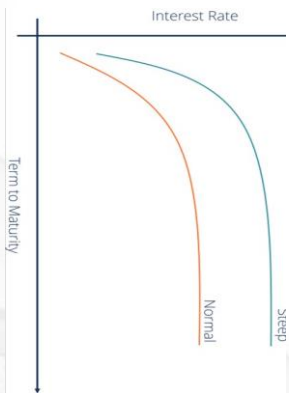
Bond Price and Time Value of Money

- Bond pricing is an application of discounted cash flow analysis.
- **Bond price** should be equal to the value of all discounted future cash flows.
- On an option-free fixed-rate bond, the promised future cash flows are a series of coupon interest payments and repayment of the full principal at maturity.
- The market discount rate is used to obtain the present value.
- The **market discount rate** is the rate of return required by investors given the risk of the investment in the bond.

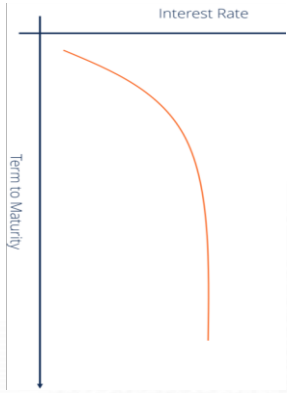
Interest rate and Term Structure

- Interest rates are both a barometer of the economy and an instrument for its control.
- The term structure of interest rates
 - ✱ Market interest rates at various maturities
 - ✱ Input for the valuation and modelling of many financial products.

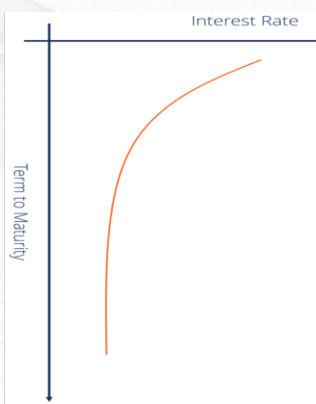
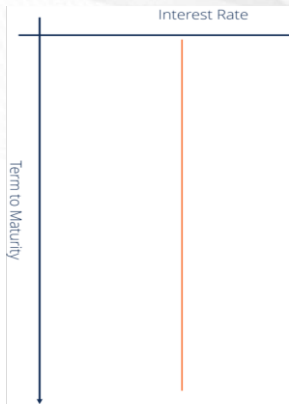
Steep Yield Curve



Normal Yield Curve



Types of Yield Curves



Theories of the Term structure of Interest Rates

Pure expectations theory

Liquidity preference theory

Traditional theories of
the term structure of
interest rates.

Segmented markets theory

Preferred habitat theory

Pure Expectations Theory

The pure expectations theory says that the forward rate is an unbiased predictor of the future spot rate.

- Its broadest interpretation suggests that investors expect the return for any investment horizon to be the same.
- The narrower interpretation—referred to as the local expectations theory—suggests that the return will be the same over a short-term horizon starting today.

Under pure expectations theory, the shape of the yield curve reflects the expectation about future short-term rates.

Liquidity Preference Theory

The **liquidity preference theory** makes the following assertion:

- Liquidity premiums exist to compensate investors for the added interest rate risk they face when lending long term, and these premiums increase with maturity.

Thus, given an expectation of unchanging short-term spot rates, liquidity preference theory predicts an upward-sloping yield curve.

Segmented market and Preferred Habitat Theory

The segmented markets theory assumes that market participants are either unwilling or unable to invest in anything other than securities of their preferred maturity.

- It follows that the yield of securities of a particular maturity is determined entirely by the supply and demand for funds of that particular maturity.

The preferred habitat theory also assumes that many borrowers and lenders have strong preferences for particular maturities.

- However, if the expected additional returns to be gained become large enough, institutions will be willing to deviate from their preferred maturities or habitats.

Yield-to-Maturity Building Blocks

Risk Free Rate of Return = Inflation Rate + Real Rate of Return

Risk Premium = Credit + Liquidity + Maturity + Tax

YTM of a Corporate Bond = Risk Free Rate + Risk Premium

Example: Inflation: 3%, Real Rate: 2%, Credit: 2%, Liquidity: 0.5%, Maturity: 0.5%, Tax: 2%, Compute the Yield (Interest Rate) on the Bond

$$\begin{aligned}
 \text{YTM} &= (\text{Risk Free Rate}) + (\text{Risk Premium}) \\
 &= (3+2) \quad + \quad (2+0.5+0.5+2) \quad = 10\%
 \end{aligned}$$

Interest Rate Risk on Bonds

Underlying Risk on Returns for Bonds

There are two offsetting types of interest rate risk

Coupon
reinvestment risk

Market price risk

Properties of Bond Duration

The duration of a bond measures the sensitivity of the bond's full price (including accrued interest) to changes in the bond's yield-to-maturity or, more generally, to changes in benchmark interest rates.

The duration for a fixed-rate bond is a function of these input variables.

- Coupon rate or payment per period
- Yield-to-maturity per period
- Time-to-maturity (as of the beginning of the period)
- Fraction of the period that has gone by
- Presence and nature of embedded options

Yield Duration Statistics

Yield duration statistics
includes

- Macaulay duration
- Modified duration
- Price value of a basis point (PVBP)

The Macaulay duration (D) formula (for the period)

Macaulay Duration - An example

A 6% annual payment bond matures on 14 February 2020 and is purchased for settlement on 11 April 2014. The YTM is 4%, following is the example to calculate Duration

Period	Time to Receipt	CF (cash flow)	PV of CF	Time-Weighted PV of CF
1	$309/365 = 0.8466$	6	$6/(1 + 0.04)^{0.8466} = 5.80$	$0.8466 \times 5.80 = 4.91$
2	1.8466	6	5.58	10.31
3	2.8466	6	5.37	15.28
4	3.8466	6	5.16	19.85
5	4.8466	6	4.96	24.05
6	5.8466	106	84.28	492.74
			111.15	567.13

$$D = 567.13/111.15 = 5.1$$

years

Approx. Modified Duration: Alternative Approach

$$AMD = \frac{(PV_- - (PV_+))}{2 \times (\Delta Yield) \times (PV_0)}$$

where PV_0 is the price of the bond at the current yield, PV_+ is the price of the bond if the yield increases (by $\Delta Yield$), and PV_- is the price of the bond if the yield decreases (by $\Delta Yield$).

Example: Consider a 6% semiannual coupon paying bond with 4 years to maturity currently priced at par (YTM = 6%).

If the YTM increases/decreases by annualized 20 bps, the price raises/decreases to 99.301 and 100.705, respectively:

$$AMD = \frac{(100.705) - (99.301)}{2 \times (0.002) \times (100)} = 3.51 \text{ Years}$$

Modified Duration

Modified duration (MD) is a direct measure of the interest rate sensitivity of a bond. It assumes that yield changes do not change the expected cash flows.



$$MD = \frac{D}{1+r}$$

where r is the yield per period.

Modified duration provides a linear estimate of the percentage price change for a bond given a change in its yield-to-maturity.



$$\% \Delta PV^{Full} \approx -MD \times \Delta Yield(\%)$$

- Note: MD is expressed in annual terms.
- To get the % change in bond price, the % change must be multiplied by the original bond price.

Bond Convexity

- Convexity is a second-order measure of interest- rate risk;
- Measures the curvature of the present value/yield profile.
- Convexity can be regarded as an indication of the error we make when using duration and modified duration, as it measures the degree to which the curvature of a bond's price/yield relationship diverges from the straight-line estimation.

Convexity

$$C = \frac{d^2 (B (r))}{B * d * r^2}$$

where:

C = convexity

B = the bond price

r = the interest rate

d = duration

Bond Convexity : Example

Bond Convexity

Example: A 6% annual payment bond matures on 14 February 2020 and is purchased for settlement on 11 April 2014. The YTM is 4%. Calculate the bond's convexity (actual/actual convention):

Period	Time to Receipt	CF	PV of CF	t^2+t	$(t^2+t) \times PV \text{ of CF}$
1	0.8466	6	5.80	1.56	9.07
2	1.8466	6	5.58	5.26	29.34
3	2.8466	6	5.37	10.95	58.76
4	3.8466	6	5.16	18.64	96.19
5	4.8466	6	4.96	28.34	140.58
6	5.8466	106	84.28	40.03	3373.63
			111.15		3707.57
$\text{Conv} = 1/(1 + 0.04)^2 \times 3707.57/111.15 = 30.84$					

Bond Convexity Approximation

Bond Convexity

Like modified duration, convexity can be accurately approximated.

- The **approximate convexity** is calculated by the following:

$$AConv = \frac{(PV_-) + (PV_+) - 2 \times (PV_0)}{(\Delta Yield)^2 \times (PV_0)}$$

Bond Convexity Approximation: Example

Example: Consider a 6% semiannual coupon paying bond with 4 years to maturity that is currently priced at par (YTM = 6%) and has an AMD of 3.51 years. If the YTM increases/ decreases by 20 bps, the price raises/decreases to 99.301 and 100.705, respectively. Calculate AConv and the effect of a 50 bps change in yield on the bond price:

$$AConv = \frac{(100.705) + (99.301) - 2 \times (100)}{(0.002)^2 \times (100)} = 14.81$$

$$\% \Delta PV^{Full} \approx -3.51 \times 0.005 + \frac{1}{2} \times 14.81 \times (0.005)^2 = 1.77\%$$

a 0.0185% convexity including adjustment

Fixed Income Portfolio Management

Key Learning Outcome

- Portfolio Management
- Immunization

Fixed Income Portfolio Management: Framework

Investor Type

Investor With Liability

The investor with liabilities will measure success by whether the portfolio generates the funds necessary to pay out the cash outflows associated with the liabilities.

Investor Without Liability

- An investor select a specific bond market index as the benchmark for the portfolio.
- The portfolio's objective is to either match or exceed the rate of return on that index.
- The risk objective of bond holdings is set not only in relation to the benchmark index but also in relation to the contribution to the risk of the overall portfolio.

Interest rate risk and the Investment Horizon

An important aspect in understanding the interest rate risk and return characteristics of an investment in a fixed-rate bond is the **time horizon**.

Bond duration is the primary measure of risk arising from a change in the yield-to-maturity; convexity is the secondary risk measure.

The common assumption in interest rate risk analysis is a parallel shift in the yield curve. In reality, the shape of the yield curve changes based on factors affecting the supply and demand of shorter- term versus longer-term securities.

Fund Management Against Liability

In managing funds against liabilities, dedication strategies are specialized fixed-income strategies that are designed to accommodate specific funding needs of the investor.

Immunization

Cash flow matching



- Immunization is a popular strategy for a guaranteed rate of return over a particular time horizon.
- The purpose of immunization is to identify the portfolio for which the change in price is exactly equal to the change in reinvestment income at the time horizon of interest.

Dedication Strategy - Immunization

Classical immunization can be defined as the creation of a fixed-income portfolio that produces an assured return for a specific time horizon, irrespective of any parallel shifts in the yield curve.



Setting the duration of the portfolio equal to specified portfolio time horizon assures the offsetting of positive and negative incremental return sources under certain assumptions, including the assumption that the immunizing portfolio has the same present value as the liability being immunized.

- An immunization strategy should be dynamic (through rebalancing) because the duration of the portfolio changes as the market yield changes and with the passage of time.
- In the immunization strategy, the investor's goal might be to reestablish the dollar duration of a portfolio to a desired level.



The diagram illustrates the formula for Dollar Duration. It consists of four circles connected by mathematical symbols. From left to right: a light green circle containing the text "Dollar duration", followed by a light green equals sign, a teal circle containing the text "Duration", followed by a light green multiplication sign, a light green circle containing the text "Portfolio value", followed by another light green multiplication sign, and finally a dark green circle containing the text "0.01".

$$\text{Dollar duration} = \text{Duration} \times \text{Portfolio value} \times 0.01$$

- A portfolio's dollar duration is equal to the sum of the dollar durations of the component securities.

Rebalancing steps

- 1) Move forward in time and include a shift in the yield curve. Using the new market values and durations, calculate the dollar duration of the portfolio at this point in time.
- 2) Calculate the rebalancing ratio by dividing the desired dollar duration by the new dollar duration. If we subtract one from this ratio and convert the result to a percent, it tells us the percentage amount that each position needs to be changed in order to rebalance the portfolio.
- 3) Multiply the new market value of the portfolio by the desired percentage change in Step 2. This number is the amount of cash needed for rebalancing.

Rebalancing based on the dollar duration:

Example. The initial portfolio of three bonds with Rs.10,00,000 par value each had a dollar duration of Rs. 111,945. After a shift in the interest rate, the portfolio values are as follows:

Security	Price	Market Value	Duration	Dollar Duration
Bond A	99.822	10,23,704	4.246	43,466
Bond B	98.728	10,04,770	0.305	3,065
Bond C	99.840	10,02,458	3.596	36,048
				82,579

- Calculate the rebalancing ratio and cash required for rebalancing to maintain dollar duration at the initial level.

Example (continued).

1) To calculate the rebalancing ratio:

$$\frac{111945}{82579} = 1.356$$

- Rebalancing requires each position to be increased by 35.6%.

2) To calculate the cash required for this rebalancing:

$$0.356 \times (1023704 + 1004770 + 1002458) = 1079012$$

Key Takeaway

Important elements to consider when investing in a fixed-income security

- the bond's features
- the legal, regulatory, and tax considerations
- The contingency provisions

Coupon Payment Structures:

fixed-coupon bonds

floating rate notes

bonds with step-up

coupons bonds with

credit-linked coupons

bonds with deferred

coupons

Common types of bonds with embedded options include callable bonds, puttable bonds, and convertible bonds.

The duration of a bond portfolio can be calculated in two ways: (1) the weighted average of the time to receipt of *aggregate* cash flows and (2) the weighted average of the durations of individual bonds that compose the portfolio.

The second method is simpler to use and quite accurate when the yield curve is relatively flat. Its main limitation is that it assumes a parallel shift in the yield curve in that the yields on all bonds in the portfolio change by the same amount

The price value of a basis point (PVBP) is an estimate of the change in the full price of a bond given a 1 bp change in the yield-to-maturity.

Effective convexity is the second-order effect on a bond price given a change in the benchmark yield curve.

Term structure of Interest Rates: The expectations theory, the liquidity preference theory and preferred habitat theory

Further Reading

- The Handbook of Fixed Income Securities: Frank J. Fabozzi
- Duration Convexity : <https://www.investopedia.com/articles/bonds/08/duration-convexity.asp>
- Benchmark : <https://www.crisil.com/en/home/what-we-do/financial-products/indices.html>
- NSE Market Data: <https://www.nseindia.com/market-data/bonds-traded-in-capital-market>
- Zero Coupon Yield Curve : https://www.ccilindia.com/RiskManagement/SecuritiesSegment/Pages/Z_CYCA.aspx
- Fixed Income Settlement : <https://www.ccilindia.com/FAQ/Risk%20Management/Pages/SecuritiesSettlement.aspx>

Thank You