Problem Set: Annuities and Perpetuities

(Solutions Below)

1. If you plan to save $300 annually for 10 years and the discount rate is 15%, what is the future value?
2. If you want to buy a boat in 6 years that costs $1,000 and you can save $150 per year, what interest rate would you need?
3. If you invest $1,000 per year in a stock portfolio with a return of 8%, how much would you expect to have in 7 years?
4. How long would it take you to save $1,000 if you invested $200 per year, and the interest rate is 10%?
5. If you need $10,000 to pay for your first year of graduate school in 3 years and you get an interest rate of 9%, how much must you invest each of the next three years?
6. If 6 years ago you invested $500 and received an interest rate of 4% (compounded monthly), how much would you now have?
7. You borrowed $100 from a friend, who said you need to pay back $300 in 5 years, what rate are you being charged if it is compounded weekly?
8. How many years would it take you to have $2,500 if you saved $100 each month at 15%?

9. To have $6000 in 7 years what interest rate would you need if you saved $200 every quarter?

10. If you win a lottery worth $1,000,000 payable in 15 years and the interest rate is 8% (compounded annually), what is this worth today? Compounded quarterly? Compounded monthly? Compounded weekly?

11. How long does it take for an investment to quadruple in value if the investment yields 6% per year (compounded monthly)?

12. What are the payments on a $40,000 loan repaid monthly for six year (r = 7%)?

13. If I invest $100 today and every quarter for 3 years in an account earning 11%, how much will I have at the end of three years?

14. Suppose that I am trying to borrow money from you to finance my business, and I promise to repay you $1,000 quarterly for two years. If your opportunity cost of funds is 10%, how much are you willing to lend me?

15. Jim makes a deposit of $120 every week (beginning next week). The deposit is to earn interest annually at the rate of 9 percent. How much will Jim have on deposit at the end of seven years?

16. How long will it take to repay a loan of $150, if I pay $1 per week and the rate on my loan is 4%?

17. Value an annuity of $300 per month for 7 years (r = 12.3%).

18. Suppose you have the opportunity to make an investment expects to pay investors $7,000 per year for next eight years. If the cost is $50,000, what return would you receive?

19. If a two year weekly annuity is worth $5000 and r = 9.8%, what is the weekly cash flow?

20. Which grows to a larger future value, $1000 invested for 2 years a) at 10 percent compounded weekly, or b) at 11 percent compounded semi-annually?

21. Value an annuity of $40 per year for ten years (r = 13%).

22. You want to save for your retirement in 50 years. How much do you need to save from your biweekly paycheck to have $5 million if you expect a return is 7%?

23. If an investment is expected to pay $400 per month for the next 14 months, how much should you be willing to pay for that asset if your cost of capital is 8%?

24. You have borrowed $35,000 at an interest rate of 9%. If you plan to pay the loan off in annual installments of $4,000 (beginning next year), when can you pay back the loan?

25. The type of house you would like to buy requires a down-payment of $50,000. You plan to make that down- payment six years from now. How much do you need to save per week (beginning next week), if your money gets 7% (annually)?

26. You hope to go to graduate school, and the tuition will be $50,000 for the one-year M.B.A. program. If you can only afford to save $3,000/quarter and the interest rate is 9%, how long will you need to save?

27. The house you plan to buy will require a down- payment of $40,000 in two years. How much do you need to save per month (beginning next month), if your savings gets 8% (annually)?

28. You have borrowed $10,000 at an interest rate of 8.7%. If you plan to pay the loan off in quarterly installments of $1,000 (beginning next quarter), how long will it take you to pay back the loan?

29. Value a perpetuity of $400 per year (r = 14.9%).

30. If a perpetuity is worth $1,000 and r = 15.5%, what is the cash flow?

Solutions

1. If you plan to save $300 annually for 10 years and the discount rate is 15%, what is the future value?

   P/Y = 1; N = 10; I/Y = 15; PV = 0; PMT = -300; FV = **$6,091.12**

2. If you want to buy a boat in 6 years that costs $1,000 and you can save $150 per year, what interest rate would you need?

   P/Y = 1; N = 6; I/Y = **4.20%**; PV = 0; PMT = -150; FV = 1,000

3. If you invest $1,000 per year in a stock portfolio with a return of 8%, how much would you expect to have in 7 years?

   P/Y = 1; N = 7; I/Y = 8; PV = 0; PMT = -1,000; FV = **$8,922.80**

4. How long would it take you to save $1,000 if you invested $200 per year, and the interest rate is 10%?

   P/Y = 1; N = **4.25 years**; I/Y = 10; PV = 0; PMT = 200; FV = - 1,000

   0.25 x 12 = 3 □ **4 years, 3 months**

   NOTE: When the question involves time, you must convert the answer to 'x years and y units'.

5. If you need $10,000 to pay for your first year of graduate school in 3 years and you get an interest rate of 9%, how much must you invest each of the next three years?

   P/Y = 1; N = 3; I/Y = 9; PV = 0; PMT = **$3,050.55**; FV = -10,000

6. If 6 years ago you invested $500 and received an interest rate of 4% (compounded monthly), how much would you now have?

P/Y = 12; N = 72 (= 6 x 12); I/Y = 4; PV = -500; PMT = 0; FV = **$635.37**

7. You borrowed $100 from a friend, who said you need to pay back $300 in 5 years, what rate are you being charged if it is compounded weekly?

   P/Y = 52; N = 260 (= 5 x 52); I/Y = **22.02%**; PV = -100; PMT = 0; FV = 300

8. How many years would it take you to have $2,500 if you saved $100 each month at 15%?

   P/Y = 12; N = **21.89 months**; I/Y = 15; PV = 0; PMT = -100; FV = 2,500

   21.89 ≈ 22 months ⇨ **1 year, 10 months**

   NOTE: Since N is periods, the time unit is the payment period.

9. To have $6000 in 7 years what interest rate would you need if you saved $200 every quarter?

   P/Y = 4; N = 28 (= 7 x 4); I/Y = **2.02%**; PV = 0; PMT = -200; FV = 6,000

10. If you win a lottery worth $1,000,000 payable in 15 years and the interest rate is 8% (compounded annually), what is this worth today? Compounded quarterly? Compounded monthly? Compounded weekly?

    P/Y = 1; N = 15; I/Y = 8; PV = **$315,241.70**; PMT = 0; FV = -1,000,000

    P/Y = 4; N = 60 (= 15 x 4); I/Y = 8; PV = **$304,782.27**; PMT = 0; FV = -1,000,000

    P/Y = 12; N = 180 (=15 x 12); I/Y = 8; PV = **$302,396.05**; PMT = 0; FV = -1,000,000

P/Y = 52; N = 780 (= 15 x 52); I/Y = 8; PV = **$301,472.08**; PMT = 0; FV = -1,000,000

11. How long does it take for an investment to quadruple in value if the investment yields 6% per year (compounded monthly)?

    P/Y = 12; N = **277.95 months**; I/Y = 6; PV = -1; PMT = 0; FV = 4

    277.95 ≈ 278 months ⇨ **23 years, 2 months**

12. What are the payments on a $40,000 loan repaid monthly for six year (r = 7%)?

    P/Y = 12; N = 72 (= 6 x 12); I/Y = 7; PV = -40,000; PMT = **$681.96**; FV = 0

13. If I invest $100 today and every quarter for 3 years in an account earning 11%, how much will I have at the end of five years?

    P/Y = 4; N = 12 (= 3 x 4); I/Y = 11; PV = 0; PMT = -100; FV = **$1,399.21**

    Value = 1,399.21 + $100 = **$1,499.21**

    You add $100 to account for the first payment coming now instead of one week from now.

14. Suppose that I am trying to borrow money from you to finance my business, and I promise to repay you $1,000 quarterly for two years. If your opportunity cost of funds is 10%, how much are you willing to lend me?

    P/Y = 4; N = 8 (= 2 x 4); I/Y = 10; PV = **$7,170.14**; PMT = -1,000; FV = 0

15. Jim makes a deposit of $120 every week (beginning next week). The deposit is to earn interest annually at the

rate of 9 percent. How much will Jim have on deposit at the end of seven years?

P/Y = 52; N = 364 (= 7 x 52); I/Y = 9; PV = 0; PMT = -120; FV = **$60,776.79**

16. How long will it take to repay a loan of $150, if I pay $1 per week and the rate on my loan is 4%?

P/Y = 52; N = **159.44**; I/Y = 4; PV = -150; PMT = 1; FV = 0

159.44 ≈ 159 weeks ⇨ **3 years, 3 weeks**

17. Value an annuity of $300 per month for 7 years (r = 12.3%).

P/Y = 12; N = 84 (= 7 x 12); I/Y = 12.3; PV = **$16,841.09**; PMT = -300; FV = 0

18. Suppose you have the opportunity to make an investment expects to pay investors $7,000 per year for next eight years. If the cost is $50,000, what return would you receive?

P/Y = 1; N = 8; I/Y = **2.59%**; PV = -50,000; PMT = 7,000; FV = 0

19. If a two year weekly annuity is worth $5000 and r = 9.8%, what is the weekly cash flow?

P/Y = 52; N = 104 (= 2 x 52); I/Y = 9.8; PV = -5,000; PMT = **$52.99**; FV = 0

20. Which grows to a larger future value, $1000 invested for 2 years a) at 10 percent compounded weekly, or b) at 11 percent compounded semi-annually?

P/Y = 52; N = 104 (= 2 x 52); I/Y = 10; PV = -1,000; PMT = 0; FV = **$1,221.17**

P/Y = 2; N = 4 (= 2 x 2); I/Y = 11; PV = -1,000; PMT = 0; FV = **$1,238.82 (better)**

21. Value an annuity of $40 per year for ten years (r = 13%).

    P/Y = 1; N = 10; I/Y = 13; PV = **$217.05**; PMT = -40; FV = 0

22. You want to save for your retirement in 50 years. How much do you need to save from your biweekly paycheck to have $5 million if you expect a return is 7%?

    P/Y = 26; N = 1300 (= 50 x 26); I/Y = 7; PV = 0; PMT = **$421.20**; FV = -5,000,000

23. If an investment is expected to pay $400 per month for the next 14 months, how much should you be willing to pay for that asset if your cost of capital is 8%?

    P/Y = 12; N = 14; I/Y = 8; PV = **$5,329.68**; PMT = -400; FV = 0

24. You have borrowed $35,000 at an interest rate of 9%. If you plan to pay the loan off in annual installments of $4,000 (beginning next year), when can you pay back the loan?

    P/Y = 1; N = **17.97**; I/Y = 9; PV = -35,000; PMT = 4,000; FV = 0

    17.97 ≈ **18 years**

25. The type of house you would like to buy requires a down-payment of $50,000. You plan to make that down- payment six years from now. How much do you need to save per week (beginning next week), if your money gets 7% (annually)?

    P/Y = 52; N = 312 (= 6 x 52); I/Y = 7; PV = 0; PMT = **$129.06**; FV = -50,000

26. You hope to go to graduate school, and the tuition will be $50,000 for the one-year M.B.A. program. If you can only afford to save $3,000/quarter and the interest rate is 9%, how long will you need to save?

P/Y = 4; N = **14.31**; I/Y = 9; PV = 0; PMT = 3,000; FV = -50,000

14.31 ≈ 14 quarters ⇨ **3 years, 2 quarters or 3 years, 6 months**

27. The house you plan to buy will require a down- payment of $40,000 in two years. How much do you need to save per month (beginning next month), if your savings gets 8% (annually)?

P/Y = 12; N = 24 (= 2 x 12); I/Y = 8; PV = 0; PMT = **$1,542.42**; FV = -40,000

28. You have borrowed $10,000 at an interest rate of 8.7%. If you plan to pay the loan off in quarterly installments of $1,000 (beginning next quarter), how long will it take you to pay back the loan?

P/Y = 4; N = **11.40**; I/Y = 8.7; PV = 10,000; PMT = -1,000; FV = 0

11.40 ≈ 11 quarters ⇨ **2 years, 3 quarters or 2 years, 9 months**

29. Value a perpetuity of $400 per year (r = 14.9%).

$$Value = \frac{C}{r} = \frac{400}{0.149} = \$2684.56$$

30. If a perpetuity is worth $1,000 and r = 15.5%, what is the cash flow

$$Value = \frac{C}{r}$$
$$C = Value \times r$$
$$= 1000 \times 0.155$$
$$= \$155.00$$

Reference:

Kogod School of Business,

WASHINGTON DC